NONSTATIONARY DOPPLER EFFECT AND FREQUENCY–PHASE METHODS OF INVESTIGATION AND CONTROL

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A general theory of the nonstationary Doppler effect, which originates during wave passage through a medium whose properties vary in time, is elucidated. Relations are obtained for the frequency shift and phase difference, and numerical estimates are made in an example of ultrasonic oscillations. The advantages of methods of investigation and control based on the effect under consideration are shown.

In investigating and controlling different physicochemical and technological processes of modern production, it is necessary to know not only the magnitudes of the different physicochemical parameters of the medium being controlled in many cases, but also their rate of change in time. This is especially characteristic for new highly intensive technological processes which are being introduced, for nonstationary methods of determining the physicochemical properties of substances, etc. Information about the rate of change of the parameter being controlled in these cases is often most important and sometimes even sufficient.

However, it should be noted that information about the rate of the process is ordinarily obtained by recording the dependence of values of the parameters being controlled and differentiating this dependence with respect to time. Such a method conceals sources of error. Firstly, the operation of differentiation itself introduces an additional error when the analytic form of the functional dependence of the parameter being controlled on the time is unknown. Secondly, inertia of the transducer which yields the information about the magnitude of the parameter being controlled is manifested in rapidly varying processes. If the transducer is inserted in the medium being investigated, and is in contact with it, then by its presence it, as a rule, already distorts the parameter field of the medium being investigated. Moreover, it always possesses an intrinsic time constant, i.e., an intrinsic inertia. Hence, for a nonstationary process the transducer readings always deviate from the real values of the parameter under investigation. Questions of the thermal inertia of contact thermal detectors are examined in detail in [1–3], etc., for example. It follows from these that an exact analytic accounting of the errors in measuring nonstationary temperatures is practically impossible because of the abundance of factors which are difficult to take into account, such as the complex nature of heat exchange, the complexity of the geometry, the dynamical errors associated with the heating rate, the difficulty in estimating the thermal resistance of the contact layer, etc.

Hence, in addition to a further improvement in contact methods (miniaturization, strict theoretical accounting of errors), the development of nondestructive low-inertia methods of investigating and controlling different, particularly thermal, nonstationary processes seems promising. One such method is that which is based on the use of the nonstationary Doppler effect.

The effect mentioned is considered in [4] for the case of discrete signal propagation in a homogeneous medium. Let us examine it for continuous waves.

If a plane wave is propagated along a certain x axis in a homogeneous medium, then any phase surface will always remain plane and perpendicular to the x axis. Let us examine a certain section of the medium in the x-axis direction which is included between the coordinate x_1 and x_2, where $L = x_2 - x_1$ is the section length. Let us assume that the velocity of wave propagation in the medium depends on the time $v = v(\tau)$, where the function $v(\tau)$ is continuous and integrable.

If the phase plane of a wave with the phase φ at the time τ_1 should enter the section of the medium under consideration and should leave it at the time τ_2, then it is evidently possible to write

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$$L = \int_{\tau_1}^{\tau_2} v(\tau)\, d\tau. \tag{1}$$

Furthermore, let the phase plane with phase $\varphi + \Delta\varphi$ enter the section at the time $\tau_1 + \Delta\tau_1$ and leave it at the time $\tau_2 + \Delta\tau_2$, then analogously

$$L = \int_{\tau_1+\Delta\tau_1}^{\tau_2+\Delta\tau_2} v(\tau)\, d\tau, \tag{2}$$

and, therefore, the equality

$$\int_{\tau_1}^{\tau_2} v(\tau)\, d\tau = \int_{\tau_1+\Delta\tau_1}^{\tau_2+\Delta\tau_2} v(\tau)\, d\tau \tag{3}$$

is valid for L = const, from which there follows

$$\int_{\tau_1}^{\tau_1+\Delta\tau_1} v(\tau)\, d\tau = \int_{\tau_2}^{\tau_2+\Delta\tau_2} v(\tau)\, d\tau. \tag{4}$$

Applying the theorem of the mean, we obtain

$$v(\eta)\,\Delta\tau_1 = v(\vartheta)\,\Delta\tau_2, \tag{5}$$

where $\tau_1 \leq \eta \leq \tau_1 + \Delta\tau_1$; $\tau_2 \leq \vartheta \leq \tau_2 + \Delta\tau_2$.

We are interested in the dependence of $\Delta\tau_2$ on $\Delta\tau_1$, which characterizes the relation between periods or the frequencies of the same wave entering into and leaving from the section under consideration. We shall henceforth consider the velocity of wave propagation not to vary in time outside the section (x_1, x_2) under consideration, i.e., the frequency of the wave entering the section agrees with the frequency of the oscillations being emitted f_0 (we consider it constant), and the frequency of oscillation at the point $x = x_2$ agrees with the frequency being received by the detector, which we denote by $f(\tau)$. Let us write the obvious relationship

$$\Delta\varphi \equiv 2\pi f_0 \Delta\tau_1 = 2\pi f(\theta)\,\Delta\tau_2, \tag{6}$$

where $\tau_2 \leq \theta \leq \tau_2 + \Delta\tau_2$. We hence obtain

$$\frac{\Delta\tau_2}{\Delta\tau_1} = \frac{f_0}{f(\theta)} . \tag{7}$$

Passing to the limit as $\Delta\varphi \to 0$ (hence $\Delta\tau_1 \to 0$, $\Delta\tau_2 \to 0$), we obtain

$$\lim_{\Delta\tau_1 \to 0} \frac{\Delta\tau_2}{\Delta\tau_1} = \frac{f_0}{f(\tau_2)} . \tag{8}$$

On the other hand, it follows from relationship (5) that

$$\lim_{\Delta\tau_1 \to 0} \frac{\Delta\tau_2}{\Delta\tau_1} = \frac{v(\tau_1)}{v(\tau_2)} . \tag{9}$$

Therefore

$$f(\tau_2) = f_0 \frac{v(\tau_2)}{v(\tau_1)} . \tag{10}$$

The frequency shift at an arbitrary time τ equals

$$\Delta f(\tau) = f(\tau) - f_0 = f_0 \left[\frac{v(\tau)}{v(\tau - \tau_3)} - 1 \right], \tag{11}$$

where $\tau = \tau_2$, $\tau - \tau_3 = \tau_1$, $\tau_3 = \tau_2 - \tau_1$ are the times of passage of a wave intersecting the boundary $x = x_2$ at the time τ, over the section (x_1, x_2) of the medium being controlled (delay time).

Relationship (11) is exact, but inconvenient to use in practice since the instantaneous values of the wave propagation velocity must be known at the time of crossing the boundaries of the section under consideration. However, if the section dimensions are such that the wave velocity during the crossing varies insignificantly, i.e.,

$$\Delta v = v(\tau) - v(\tau - \tau_3) \ll v(\tau), \tag{12}$$

then it can be assumed with sufficient accuracy that

$$v(\tau-\tau_3)=v(\tau)-\frac{dv}{d\tau}\tau_3,\quad \tau_3=\frac{L}{v(\tau)}, \tag{13}$$

and then we finally obtain

$$\Delta f(\tau)=\frac{f_0 L}{v^2(\tau)}\,\frac{dv}{d\tau}. \tag{14}$$

As is seen from (11) and (14), the effect being described is substantially nonstationary, and due to the change in time of the properties of the medium affecting the wave propagation velocity. It must be emphasized that these relationships have been obtained under the assumption L = const, i.e., there is no ordinary (kinematic) Doppler effect.

In addition to the frequency shift described, a change in wave propagation velocity in a medium evokes the appearance of an additional phase difference equal to

$$\Delta\varphi(\tau)=2\pi\int_0^\tau \Delta f(\tau)\,d\tau=\frac{2\pi f_0\Delta v(\tau)}{v_0[v_0+\Delta v(\tau)]}, \tag{15}$$

where $\tau = 0$ is taken as the origin and v_0 is the mean velocity of wave propagation in the medium corresponding to this time (initial velocity).

Relation (15) is exact. Let us also note that (11), (14), and (15) are valid for any waves independently of their nature (elastic, electromagnetic, light, transverse, longitudinal, surface, etc.).

In the general case, the effects considered can be caused by a simultaneous change in a number of physical parameters $K_1, K_2, K_3, \ldots, K_n$ which influence the velocity of wave propagation. In this case, by considering their changes continuous, we can write

$$dv=\sum_{i=1}^{n}\frac{dv}{dK_i}\,dK_i\equiv\sum_{i=1}^{n}b_i dK_i,$$
$$\frac{dv}{d\tau}=\sum_{i=1}^{n}b_i\frac{dK_i}{d\tau}. \tag{16}$$

Let us examine the case when only one physical parameter K varies, which might be the temperature, pressure, concentration, etc. If its range of variation is slight, then dependence v(K) can be assumed linear with sufficient accuracy, i.e., $dv/dK = b = \text{const}$, and we easily obtain from (14) and (15):

$$\frac{dK}{d\tau}=\frac{v^2}{f_0 Lb}\Delta f, \tag{17}$$

$$\Delta K=\frac{v_0^2\Delta\varphi}{b(2\pi f_0 L-v_0\Delta\varphi)}. \tag{18}$$

As is seen from these last relationships, the frequency shift yields information about the rate of change of the physical parameter being controlled, while the additional shift in the phase difference yields information about the magnitude of its change. Simultaneous recording of the time dependence Δf and $\Delta\varphi$ yields complete information about the dynamics of the procedure of the process under investigation.

The resolution of methods of investigation and control based on the effect described is determined by the accuracy of measuring the frequency and phase shifts. The nonstationary Doppler effect for continuous ultrasonic oscillations of 1- and 5-MHz frequency is used [5-9]. The measuring systems were constructed by using the principle of multiplication of the frequency shift or the phase shift 10^n times, where n = 1, 2, 3,... (the principle of frequency multiplication with subsequent heterodyne conversion). The system operates reliably for a 10^4 times multiplication. The error in measuring the phase shift is hence not more than 10^{-2} degrees of angle. The error in measuring the frequency shift depends on its magnitude and the measurement times (averaging time); however, it is actually not difficult to achieve conditions under which it will not exceed 10^{-4} Hz. Taking these as initial data, let us estimate the resolution of the frequency–phase methods. Let us take the temperature as the parameter being controlled. We take $f_0 = 10^6$ Hz, L = 0.2 m, $v = 5\cdot10^3$ m/sec (metals) for the numerical estimates. The temperature coefficient of the ultrasound velocity for metals is on the order of $|dv/dT| = b = 1$ m/sec·deg K.

Substituting these data and the above-mentioned errors for Δf and $\Delta\varphi$ into (17) and (18), we obtain the minimal velocity and the magnitude of the temperature change which can be recorded thus:

$$\frac{dT}{d\tau} \approx 10^{-2} \deg \mathrm{K/sec}, \ \delta T \approx 3.5 \cdot 10^{-3} \ {}^\circ\mathrm{K}.$$

For fluids we take $v = 1.5 \cdot 10^3$ m/sec (water), $b = 3$ m/sec · deg K, then analogously

$$\frac{dT}{d\tau} \approx 4 \cdot 10^{-4} \deg \mathrm{K/sec}, \ \delta T \approx 10^{-4} \ {}^\circ\mathrm{K}.$$

For gases we take $v = 340$ m/sec (air), $b = 0.6$ m/sec · deg K, $L = 5$ cm (for $L = 0.2$ m there will be strong damping in air at the frequency $f_0 = 1$ MHz), then

$$\frac{dT}{d\tau} \approx 4 \cdot 10^{-4} \deg \mathrm{K/sec}, \delta T \approx 1.1 \cdot 10^{-4} \ {}^\circ\mathrm{K}$$

It follows from the estimates presented that extremely slow processes can be controlled by the ultrasonic frequency–phase method by recording negligible changes in the parameter being controlled. Let us note that the numbers presented are not limits. The resolution can be elevated by going over to a higher frequency of the ultrasonic oscillations being emitted in the medium or by applying a greater than 10^4 times multiplication of the frequency–phase shifts.

The upper bound of the measurements of the rate of change of the parameter being controlled is determined principally by the inertia of the acoustic channel, i.e., by the time the wave takes to pass the section of medium being controlled. For the values of v and L taken above for metals, the delay time is $\tau_3 = 4 \cdot 10^{-5}$ sec. This means that under these conditions nonstationary processes causing frequency shifts to $\Delta f_{max} = 1/\tau_3 = 2.5 \cdot 10^4$ Hz can be recorded by the method described. Performing the computation for the temperature by (17), we obtain

$$\left(\frac{dT}{d\tau}\right)_{max} \approx 3 \cdot 10^6 \deg \mathrm{K/sec}.$$

The corresponding values of $(dT/d\tau)_{max}$ for fluids and gases are approximately two orders of magnitude lower.

Therefore, use of the nonstationary Doppler effect affords the possibility of recording practically arbitrarily rapid processes, including shocks, explosions, etc.

It should certainly be kept in mind that the methods described cannot completely replace other existing methods. For instance, it is impossible to determine the nature of the change in a physical parameter at some point of the object being investigated by using them; they afford the possibility of recording the rate and magnitude of the change in just the mean value of the parameter on the section of the medium under control but with great accuracy. Furthermore, these methods permit the measurement of only excess values of the parameters, and their changes from the beginning of the measurements. The initial values must be determined by other methods. Difficulties also occur in the use of these methods for media with high absorption of the probing waves (these are porous, friable media, certain plastics, etc. for ultrasound). At the same time, they possess a number of indubitable advantages, high accuracy, low inertia, noise rejection, the broad possibility of automating the experiment, etc., which make their application and further development both urgent and promising.

NOTATION

v, wave propagation velocity in the medium; x, coordinate; τ, time; φ, phase of the oscillations; f, frequency (f_0 and f are the frequencies of the emitted and received oscillations, respectively); τ_3, delay time; T, temperature; K, $K_1, \ldots, K_n$, some physical parameters of the medium affecting the wave propagation velocity.

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SOME CHARACTERISTICS OF A PLASMATRON WITH AN INTERELECTRODE INSERT

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A critical analysis of the criterion $R_a^* + R_b > 0$ is made for a dc electric arc in the region of stable burning. The numerical values of the dynamic factor are determined and the dynamic current–voltage characteristic is investigated.

In [1, 2] the method of static current–voltage characteristics (SCVC) was used to establish the range of variation of the parameters of a dc electric arc centered by a distributed air vortex along a discharge chamber with a sectioned ($d_s = 1 \cdot 10^{-2}$ m, $n_s = 3$) interelectrode insert.

Devices of this type give a U-shaped SCVC ($R_a^* = dU_a/dI_a \gtrless 0$). This indicates [3] that current amplification in the arc is due both to the dominant increase in free-electron concentration in comparison with the reduction of their directional velocity (part with $R_a^* < 0$) and to the opposite relation of these characteristics (part with $R_a^* > 0$). In particular, in the considered plasma generator only the falling part ($R_a < 0$) of the SCVC could be obtained owing to the limitations of the power supply (U_L = 0–400 V; I_L = 200 A).

In this case, in accordance with the Kaufmann stability criterion [4]

$$K = R_a^* + R_b > 0 \tag{1}$$

a freely burning arc can exist for a long time only if the electrical circuit contains a series-connected ballast rheostat. We will analyze the validity of condition (1) for an arc operating in conditions of forced spatial stabilization. Criterion (1) is graphically illustrated in Fig. 1.

All points with coordinates belonging to the plane situated on the right of the bent line R^*–M–N will correspond to stable regimes. Since $R_a^* < 0$, the region ABC of existence of the arc lies below the R_b axis and well to the right of the interface MN($R_b = -R_a^*$). Hence, although condition (1) is satisfied on the boundaries AB and AC (the boundary BC identifies the maximum permissible prolonged current I_{max} = 200 A of the experimental device) the arc is extinguished. This fact leads to the following conclusion: In the case of an electric arc spatially stabilized by a gas vortex criterion (1) is a necessary, but not sufficient, condition for its existence. It is probably the unsteady mechanism of interaction of the positive column of the arc and its electrode regions with the surrounding medium that is responsible for the observed anomaly at the boundaries of the region ABC and for its limited size. In the discharge chamber of a plasma generator there occur numerous intense thermal, electric, and magnetohydrodynamic processes of a complex nature. In addition, the considered picture is greatly complicated by their pronounced unsteadiness.

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